Package ‘SuperGauss’

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Description
Likelihood evaluations for stationary Gaussian time series are typically obtained via the Durbin-Levinson algorithm, which scales as O(n^2) in the number of time series observations. This package provides a ``superfast'' O(n log^2 n) algorithm written in C++, crossing over with Durbin-Levinson around n = 300. Efficient implementations of the score and Hessian functions are also provided, leading to superfast versions of inference algorithms such as Newton-Raphson and Hamiltonian Monte Carlo. The C++ code provides a Toeplitz matrix class packaged as a header-only library, to simplify low-level usage in other packages and outside of R.

License GPL-3

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`SuperGauss-package` *Superfast inference for stationary Gaussian time series.*

Description

Likelihood evaluations for stationary Gaussian time series are typically obtained via the Durbin-Levinson algorithm, which scales as O(n^2) in the number of time series observations. This package provides a "superfast" O(n log^2 n) algorithm written in C++, crossing over with Durbin-Levinson around n = 300. Efficient implementations of the score and Hessian functions are also provided, leading to superfast versions of inference algorithms such as Newton-Raphson and Hamiltonian Monte Carlo. The C++ code provides a Toeplitz matrix class packaged as a header-only library, to simplify low-level usage in other packages and outside of R.

Details

While likelihood calculations with stationary Gaussian time series generally scale as `O(N^2)` in the number of observations, this package implements an algorithm which scales as O(N log^2 N). "Superfast" algorithms for loglikelihood gradients and Hessians are also provided. The underlying C++ code is distributed through a header-only library found in the installed package's `include` directory.

Author(s)

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Examples

```
# Superfast inference for the timescale parameter
# of the exponential autocorrelation function
exp_acf <- function(lambda) exp(-(1:N-1)/lambda)

# simulate data
lambda0 <- 1
N <- 1000
X <- rnormtz(n = 1, acf = exp_acf(lambda0))

# loglikelihood function
# allocate memory for a NormalToeplitz distribution object
NTz <- NormalToeplitz$new(N)
loglik <- function(lambda) {
  NTz$logdens(z = X, acf = exp_acf(lambda))
  ## dSnorm(X = X, acf = Toep, log = TRUE)
}

# maximum likelihood estimation
optimize(f = loglik, interval = c(.2, 5), maximum = TRUE)
```

`acf2incr` *Convert position autocorrelations to increment autocorrelations.*

Description

Convert the autocorrelation of a stationary sequence x = (x_1, ..., x_N) to that of its increments, dx = (x_2 - x_1, ..., x_N - x_(N-1)).

Usage

```
acf2incr(acf)
```

Arguments

`acf`	Length-`N` vector of position autocorrelations.

Value

Length `N-1` vector of increment autocorrelations.

Examples

```
acf2incr(acf = exp(-(0:10)))
```

acf2msd	*Convert autocorrelation of stationary increments to mean squared displacement of posititions.*

Description

Converts the autocorrelation of a stationary increment sequence dx = (x_1 - x_0, ..., x_N - x_(N-1)) to the mean squared displacement (MSD) of the corresponding positions, i.e., `MSD_i = E[(x_i -x_0)^2]`.

Usage

```
acf2msd(acf)
```

Arguments

`acf`	Length-`N` autocorrelation vector of a stationary increment sequence.

Value

Length-`N` MSD vector of the corresponding positions.

Examples

```
acf2msd(acf = exp(-(0:10)))
```

Cholesky	*Cholesky multiplication with Toeplitz variance matrices.*

Description

Multiplies the Cholesky decomposition of the Toeplitz matrix with another matrix, or solves a system of equations with the Cholesky factor.

Usage

```
cholZX(Z, acf)

cholXZ(X, acf)
```

Arguments

`Z`	Length-`N` or N x p matrix of residuals.
`acf`	Length-`N` autocorrelation vector of the Toeplitz variance matrix.
`X`	Length-`N` or N x p matrix of observations.

Details

If `C == t(chol(toeplitz(acf)))`, then `cholZX()` computes `C %*% Z` and `cholZX()` computes `solve(C,X)`. Both functions use the Durbin-Levinson algorithm.

Value

Size N x p residual or observation matrix.

Examples

```
N <- 10
p <- 2
acf <- exp(-(1:N - 1))

Z <- matrix(rnorm(N * p), N, p)
cholZX(Z = Z, acf = acf) - (t(chol(toeplitz(acf))) %*%  Z)

X <- matrix(rnorm(N * p), N, p)
cholXZ(X = X, acf = acf) - solve(t(chol(toeplitz(acf))), X)
```

`Circulant` *Constructor and methods for Circulant matrix objects.*

Description

Constructor and methods for Circulant matrix objects.

Methods

Public methods:

- `Circulant$new()`
- `Circulant$size()`
- `Circulant$set_acf()`
- `Circulant$get_acf()`
- `Circulant$set_psd()`
- `Circulant$get_psd()`
- `Circulant$has_acf()`
- `Circulant$prod()`
- `Circulant$solve()`
- `Circulant$log_det()`
- `Circulant$clone()`

Method `new()`**:** Class constructor.

Usage:

```
Circulant$new(N, uacf, upsd)
```

Arguments:

`N` Size of Circulant matrix.

`uacf` Optional vector of `Nu = floor(N/2)+1` unique elements of the autocorrelation.

`upsd` Optional vector of `Nu = floor(N/2)+1` unique elements of the PSD.

Returns: A `Circulant` object.

Method `size()`**:** Get the size of the Circulant matrix.

Usage:

```
Circulant$size()
```

Returns: Size of the Circulant matrix.

Method `set_acf()`**:** Set the autocorrelation of the Circulant matrix.

Usage:

```
Circulant$set_acf(uacf)
```

Arguments:

`uacf` Vector of `Nu = floor(N/2)+1` unique elements of the autocorrelation.

Method `get_acf()`**:** Get the autocorrelation of the Circulant matrix.

Usage:

```
Circulant$get_acf()
```

Returns: The complete autocorrelation vector of length `N`.

Method `set_psd()`**:** Set the PSD of the Circulant matrix.

The power spectral density (PSD) of a Circulant matrix `Ct = Circulant(acf)` is defined as `psd = iFFT(acf)`.

Usage:

```
Circulant$set_psd(upsd)
```

Arguments:

`upsd` Vector of `Nu = floor(N/2)+1` unique elements of the psd.

Method `get_psd()`**:** Get the PSD of the Circulant matrix.

Usage:

```
Circulant$get_psd()
```

Returns: The complete PSD vector of length `N`.

Method `has_acf()`**:** Check whether the autocorrelation of the Circulant matrix has been set.

Usage:

```
Circulant$has_acf()
```

Returns: Logical; `TRUE` if `Circulant$set_acf()` has been called.

Method `prod()`**:** Circulant matrix-matrix product.

Usage:

```
Circulant$prod(x)
```

Arguments:

`x` Vector or matrix with `N` rows.

Returns: The matrix product `Ct %*% x`.

Method `solve()`**:** Solve a Circulant system of equations.

Usage:

```
Circulant$solve(x)
```

Arguments:

`x` Optional vector or matrix with `N` rows.

Returns: The solution in `z` to the system of equations `Ct %*% z = x`. If `x` is missing, returns the inverse of `Ct`.

Method `log_det()`**:** Calculate the log-determinant of the Circulant matrix.

Usage:

```
Circulant$log_det()
```

Returns: The log-determinant `log(det(Ct))`.

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

```
Circulant$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

`dnormtz` *Density of a multivariate normal with Toeplitz variance matrix.*

Description

Density of a multivariate normal with Toeplitz variance matrix.

Usage

```
dnormtz(X, mu, acf, log = FALSE, method = c("gschur", "ltz"))
```

Arguments

`X`	Vector of length `N` or N x n matrix, of which each column is a multivariate observation.
`mu`	Vector or matrix of mean values of compatible dimensions with `X`. Defaults to all zeros.
`acf`	Vector of length `N` containing the first column of the Toeplitz variance matrix.
`log`	Logical; whether to return the multivariate normal density on the log scale.
`method`	Which calculation method to use. Choices are: `gschur` for a modified version of the Generalized Schur algorithm of Ammar & Gragg (1988), or `ltz` for the Levinson-Trench-Zohar method. The former scales as O(N log^2 N) whereas the latter scales as `O(N^2)` and should only be used for `N < 300`.

Value

Vector of n (log-)densities, one for each column of X.

Examples

```
# simulate data
N <- 10 # length of each time series
n <- 3 # number of time series
theta <- 0.1
lambda <- 2
mu <- theta^2 * rep(1, N)
acf <- exp(-lambda * (1:N - 1))

X <- rnormtz(n, acf = acf) + mu

# evaluate log-density
dnormtz(X, mu, acf, log = TRUE)
```

---

fbm_msd *Mean square displacement of fractional Brownian motion.*

Description

Mean square displacement of fractional Brownian motion.

Usage

```
fbm_msd(tseq, H)
```

Arguments

tseq	Length-N vector of timepoints.
H	Hurst parameter (between 0 and 1).

Details

The mean squared displacement (MSD) of a stochastic process X_t is defined as

MSD(t) = E[(X_t - X_0)^2].

Fractional Brownian motion (fBM) is a continuous Gaussian process with stationary increments, such that its covariance function is entirely defined the MSD, which in this case is MSD(t) = |t|^(2H).

Value

Length-N vector of mean square displacements.

Examples

```
fbm_msd(tseq = 1:10, H = 0.4)
```

`matern_acf` *Matern autocorrelation function.*

Description

Matern autocorrelation function.

Usage

```
matern_acf(tseq, lambda, nu)
```

Arguments

`tseq`	Vector of `N` time points at which the autocorrelation is to be calculated.
`lambda`	Timescale parameter.
`nu`	Smoothness parameter.

Details

The Matern autocorrelation is given by

$$\mathrm{ACF}(t) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu}\frac{t}{\lambda}\right)^{\nu} K_\nu\left(\sqrt{2\nu}\frac{t}{\lambda}\right),$$

where $K_\nu(x)$ is the modified Bessel function of second kind.

Value

An autocorrelation vector of length `N`.

Examples

```
matern_acf(tseq = 1:10, lambda = 1, nu = 3/2)
```

`msd2acf` *Convert mean square displacement of positions to autocorrelation of increments.*

Description

Converts the mean squared displacement (MSD) of a stationary increments sequence x = (x_0, x_1, ..., x_N) positions to the autocorrelation of the corresponding increments dx = (x_1 - x_0, ..., x_N - x_(N-1)).

Usage

```
msd2acf(msd)
```

Arguments

msd — Length-N MSD vector, i.e., excluding x_0 which is assumed to be zero.

Value

Length-N autocorrelation vector.

Examples

```
# autocorrelation of fBM increments
msd2acf(msd = fbm_msd(tseq = 0:10, H = .3))
```

NormalCirculant — *Multivariate normal with Circulant variance matrix.*

Description

Provides methods for the Normal-Circulant (NCt) distribution, which for a random vector z of length N is defined as

```
z ~ NCt(uacf)   <=>   z ~ Normal(0, toeplitz(acf)),
```

where uacf are the Nu = floor(N/2)+1 unique elements of the autocorrelation vector acf, i.e.,

```
acf = (uacf, rev(uacf[2:(Nu-1)]),   N even,
    = (uacf, rev(uacf[2:Nu])),      N odd.
```

Methods

Public methods:

- NormalCirculant$new()
- NormalCirculant$size()
- NormalCirculant$logdens()
- NormalCirculant$grad_full()
- NormalCirculant$clone()

Method new()**:** Class constructor.

Usage:

```
NormalCirculant$new(N)
```

Arguments:

N Size of the NCt random vector.

Returns: A NormalCirculant object.

Method size()**:** Get the size of the NCt random vector.

Usage:

`NormalCirculant$size()`

Returns: Size of the NCt random vector.

Method `logdens()`**:** Log-density function.

Usage:

`NormalCirculant$logdens(z, uacf)`

Arguments:

`z` Density argument. A vector of length `N` or an N x n_obs matrix where each column is an `N`-dimensional observation.

`uacf` A vector of length `Nu = floor(N/2)` containing the first half of the autocorrelation (i.e., first row/column) of the Circulant variance matrix.

Returns: A scalar or vector of length `n_obs` containing the log-density of the NCt evaluated at its arguments.

Method `grad_full()`**:** Full gradient of log-density function.

Usage:

`NormalCirculant$grad_full(z, uacf, calc_dldz = TRUE, calc_dldu = TRUE)`

Arguments:

`z` Density argument. A vector of length `N`.

`uacf` A vector of length `Nu = floor(N/2)` containing the first half of the autocorrelation (i.e., first row/column) of the Circulant variance matrix.

`calc_dldz` Whether or not to calculate the gradient with respect to `z`.

`calc_dldu` Whether or not to calculate the gradient with respect to `uacf`.

Returns: A list with elements:

`ldens` The log-density evaluated at `z` and `uacf`.

`dldz` The length-`N` gradient vector with respect to `z`, if `calc_dldz = TRUE`.

`dldu` The length-`Nu = floor(N/2)+1` gradient vector with respect to `uacf`, if `calc_dldu = TRUE`.

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

`NormalCirculant$clone(deep = FALSE)`

Arguments:

`deep` Whether to make a deep clone.

`NormalToeplitz` *Multivariate normal with Toeplitz variance matrix.*

Description

Provides methods for the Normal-Toeplitz (NTz) distribution defined as

```
z ~ NTz(acf)   <=>   z ~ Normal(0, toeplitz(acf)),
```

i.e., for a multivariate normal with mean zero and variance `Tz = toeplitz(acf)`.

Methods

Public methods:

- `NormalToeplitz$new()`
- `NormalToeplitz$size()`
- `NormalToeplitz$logdens()`
- `NormalToeplitz$grad()`
- `NormalToeplitz$hess()`
- `NormalToeplitz$grad_full()`
- `NormalToeplitz$clone()`

Method `new()`: Class constructor.

Usage:

```
NormalToeplitz$new(N)
```

Arguments:

`N` Size of the NTz random vector.

Returns: A `NormalToeplitz` object.

Method `size()`: Get the size of the NTz random vector.

Usage:

```
NormalToeplitz$size()
```

Returns: Size of the NTz random vector.

Method `logdens()`: Log-density function.

Usage:

```
NormalToeplitz$logdens(z, acf)
```

Arguments:

`z` Density argument. A vector of length `N` or an N x n_obs matrix where each column is an `N`-dimensional observation.

`acf` A vector of length `N` containing the autocorrelation (i.e., first row/column) of the Toeplitz variance matrix.

Returns: A scalar or vector of length `n_obs` containing the log-density of the NTz evaluated at its arguments.

Method `grad()`**:** Gradient of the log-density with respect to parameters.

Usage:

```
NormalToeplitz$grad(z, dz, acf, dacf, full_out = FALSE)
```

Arguments:

`z` Density argument. A vector of length `N`.

`dz` An N x n_theta matrix containing the gradient `dz/dtheta`.

`acf` A vector of length `N` containing the autocorrelation of the Toeplitz variance matrix.

`dacf` An N x n_theta matrix containing the gradient `dacf/dtheta`.

`full_out` If `TRUE`, returns the log-density as well (see 'Value').

Returns: A vector of length `n_theta` containing the gradient of the NTz log-density with respect to `theta`, or a list with elements `ldens` and `grad` consisting of the log-density and the gradient vector.

Method `hess()`**:** Hessian of log-density with respect to parameters.

Usage:

```
NormalToeplitz$hess(z, dz, d2z, acf, dacf, d2acf, full_out = FALSE)
```

Arguments:

`z` Density argument. A vector of length `N`.

`dz` An N x n_theta matrix containing the gradient `dz/dtheta`.

`d2z` An N x n_theta x n_theta array containing the Hessian d^2z/dtheta^2.

`acf` A vector of length `N` containing the autocorrelation of the Toeplitz variance matrix.

`dacf` An N x n_theta matrix containing the gradient `dacf/dtheta`.

`d2acf` An N x n_theta x n_theta array containing the Hessian `dacf^2/dtheta^2`.

`full_out` If `TRUE`, returns the log-density and its gradient as well (see 'Value').

Returns: An n_theta x n_theta matrix containing the Hessian of the NTz log-density with respect to `theta`, or a list with elements `ldens`, `grad`, and `hess` consisting of the log-density, its gradient (a vector of size `n_theta`), and the Hessian matrix, respectively.

Method `grad_full()`**:** Full gradient of log-density function.

Usage:

```
NormalToeplitz$grad_full(z, acf, calc_dldz = TRUE, calc_dlda = TRUE)
```

Arguments:

`z` Density argument. A vector of length `N`.

`acf` A vector of length `N` containing the autocorrelation of the Toeplitz variance matrix.

`calc_dldz` Whether or not to calculate the gradient with respect to `z`.

`calc_dlda` Whether or not to calculate the gradient with respect to `acf`.

Returns: A list with elements:

`ldens` The log-density evaluated at `z` and `acf`.

`dldz` The length-`N` gradient vector with respect to `z`, if `calc_dldz = TRUE`.

`dlda` The length-`N` gradient vector with respect to `acf`, if `calc_dlda = TRUE`.

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

```
NormalToeplitz$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

`pex_acf` *Power-exponential autocorrelation function.*

Description

Power-exponential autocorrelation function.

Usage

```
pex_acf(tseq, lambda, rho)
```

Arguments

`tseq`	Vector of `N` time points at which the autocorrelation is to be calculated.
`lambda`	Timescale parameter.
`rho`	Power parameter.

Details

The power-exponential autocorrelation function is given by:

$$\mathrm{ACF}(t) = \exp\left\{-(t/\lambda)^\rho\right\}.$$

Value

An autocorrelation vector of length `N`.

Examples

```
pex_acf(tseq = 1:10, lambda = 1, rho = 2)
```

`rnormtz` *Simulate a stationary Gaussian time series.*

Description

Simulate a stationary Gaussian time series.

Usage

```
rnormtz(n = 1, acf, Z, fft = TRUE, nkeep, tol = 1e-06)
```

Arguments

`n`	Number of time series to generate.
`acf`	Length-`N` vector giving the autocorrelation of the series.
`Z`	Optional size (2N-2) x n or N x n matrix of iid standard normals, to use in the FFT and Durbin-Levinson methods, respectively.
`fft`	Logical; whether or not to use the O(N log N) FFT-based algorithm of Wood and Chan (1994) or the more stable `O(N^2)` Durbin-Levinson algorithm. See Details.
`nkeep`	Length of time series. Defaults to `N = length(acf)`. See Details.
`tol`	Relative tolerance on negative eigenvalues. See Details.

Details

The FFT method fails when the embedding circulant matrix is not positive definite. This is typically due to one of two things:

1. Roundoff error can make tiny eigenvalues appear negative. For this purpose, argument `tol` can be used to replace all negative eigenvalues by `tol * ev_max`, where `ev_max` is the largest eigenvalue.
2. The autocorrelation is decaying too slowly on the given timescale. To mitigate this, argument `nkeep` can be used to supply a longer `acf` than is required, and keep only the first `nkeep` time series observations. For consistency, `nkeep` also applies to Durbin-Levinson method.

Value

Length-`nkeep` vector or size nkeep x n matrix with time series as columns.

Examples

```
N <- 10
acf <- exp(-(1:N - 1)/N)
rnormtz(n = 3, acf = acf)
```

`SuperGauss-defunct` *Defunct functions in* **SuperGauss**.

Description

Defunct functions in **SuperGauss**.

The following functions have been removed from the SuperGauss package

`rSnorm()` Please use `rnormtz()` instead.

`dSnorm()` Please use `dnormtz()` instead.

`Snorm.grad()` Please use the `grad()` method in the NormalToeplitz class.

`Snorm.hess()` Please use the `hess()` method in the NormalToeplitz class.

`toep.mult` *Toeplitz matrix multiplication.*

Description

Efficient matrix multiplication with Toeplitz matrix and arbitrary matrix or vector.

Usage

```
toep.mult(acf, X)
```

Arguments

`acf`	Length-`N` vector giving the first column (or row) of the Toeplitz matrix.
`X`	Vector or matrix of compatible dimensions with `acf`.

Value

An `N`-row matrix corresponding to `toeplitz(acf) %*% X`.

Examples

```
N <- 20
d <- 3
acf <- exp(-(1:N))
X <- matrix(rnorm(N*d), N, d)
toep.mult(acf, X)
```

Toeplitz *Constructor and methods for Toeplitz matrix objects.*

Description

The Toeplitz class contains efficient methods for linear algebra with symmetric positive definite (i.e., variance) Toeplitz matrices.

Usage

```
is.Toeplitz(x)

as.Toeplitz(x)

## S3 method for class 'Toeplitz'
dim(x)
```

Arguments

x	An R object.

Details

An N x N Toeplitz matrix Tz is defined by its length-N "autocorrelation" vector acf, i.e., first row/column Tz. Thus, for the function stats::toeplitz(), we have Tz = toeplitz(acf).

It is assumed that acf defines a valid (i.e., positive definite) variance matrix. The matrix multiplication methods still work when this is not the case but the other methods do not (return values typically contain NaNs).

as.Toeplitz(x) attempts to convert its argument to a Toeplitz object by calling Toeplitz$new(acf = x). is.Toeplitz(x) checks whether its argument is a Toeplitz object.

Methods

Public methods:

- Toeplitz$new()
- Toeplitz$print()
- Toeplitz$size()
- Toeplitz$set_acf()
- Toeplitz$get_acf()
- Toeplitz$has_acf()
- Toeplitz$prod()
- Toeplitz$solve()
- Toeplitz$log_det()
- Toeplitz$trace_grad()
- Toeplitz$trace_hess()

- Toeplitz$clone()

Method `new()`**:** Class constructor.

Usage:

`Toeplitz$new(N, acf)`

Arguments:

`N` Size of Toeplitz matrix.

`acf` Autocorrelation vector of length `N`.

Returns: A `Toeplitz` object.

Method `print()`**:** Print method.

Usage:

`Toeplitz$print()`

Method `size()`**:** Get the size of the Toeplitz matrix.

Usage:

`Toeplitz$size()`

Returns: Size of the Toeplitz matrix. `ncol()`, `nrow()`, and `dim()` methods for `Toeplitz` objects also work as expected.

Method `set_acf()`**:** Set the autocorrelation of the Toeplitz matrix.

Usage:

`Toeplitz$set_acf(acf)`

Arguments:

`acf` Autocorrelation vector of length `N`.

Method `get_acf()`**:** Get the autocorrelation of the Toeplitz matrix.

Usage:

`Toeplitz$get_acf()`

Returns: The autocorrelation vector of length `N`.

Method `has_acf()`**:** Check whether the autocorrelation of the Toeplitz matrix has been set.

Usage:

`Toeplitz$has_acf()`

Returns: Logical; `TRUE` if `Toeplitz$set_acf()` has been called.

Method `prod()`**:** Toeplitz matrix-matrix product.

Usage:

`Toeplitz$prod(x)`

Arguments:

`x` Vector or matrix with `N` rows.

Returns: The matrix product `Tz %*% x`. `Tz %*% x` and `x %*% Tz` also work as expected.

Method `solve()`**:** Solve a Toeplitz system of equations.

Usage:

```
Toeplitz$solve(x, method = c("gschur", "pcg"), tol = 1e-10)
```

Arguments:

`x` Optional vector or matrix with `N` rows.

`method` Solve method to use. Choices are: `gschur` for a modified version of the Generalized Schur algorithm of Ammar & Gragg (1988), or `pcg` for the preconditioned conjugate gradient method of Chen et al (2006). The former is faster and obtains the log-determinant as a direct biproduct. The latter is more numerically stable for long-memory autocorrelations.

`tol` Tolerance level for the `pcg` method.

Returns: The solution in `z` to the system of equations `Tz %*% z = x`. If `x` is missing, returns the inverse of `Tz`. `solve(Tz,x)` and `solve(Tz,x,method,tol)` also work as expected.

Method `log_det()`**:** Calculate the log-determinant of the Toeplitz matrix.

Usage:

```
Toeplitz$log_det()
```

Returns: The log-determinant `log(det(Tz))`. `determinant(Tz)` also works as expected.

Method `trace_grad()`**:** Computes the trace-gradient with respect to Toeplitz matrices.

Usage:

```
Toeplitz$trace_grad(acf2)
```

Arguments:

`acf2` Length-`N` autocorrelation vector of the second Toeplitz matrix. This matrix must be symmetric but not necessarily positive definite.

Returns: Computes the trace of

`solve(Tz, toeplitz(acf2))`.

This is used in the computation of the gradient of `log(det(Tz(theta)))` with respect to `theta`.

Method `trace_hess()`**:** Computes the trace-Hessian with respect to Toeplitz matrices.

Usage:

```
Toeplitz$trace_hess(acf2, acf3)
```

Arguments:

`acf2` Length-`N` autocorrelation vector of the second Toeplitz matrix. This matrix must be symmetric but not necessarily positive definite.

`acf3` Length-`N` autocorrelation vector of the third Toeplitz matrix. This matrix must be symmetric but not necessarily positive definite.

Returns: Computes the trace of

`solve(Tz, toeplitz(acf2)) %*% solve(Tz, toeplitz(acf3))`.

This is used in the computation of the Hessian of `log(det(Tz(theta)))` with respect to `theta`.

Method `clone()`**:** The objects of this class are cloneable with this method.

Usage:

```
Toeplitz$clone(deep = FALSE)
```

Arguments:

`deep` Whether to make a deep clone.

Examples

```
# construct a Toeplitz matrix
acf <- exp(-(1:5))
Tz <- Toeplitz$new(acf = acf)
# alternatively, can allocate space first
Tz <- Toeplitz$new(N = length(acf))
Tz$set_acf(acf = acf)

# basic methods
Tz$get_acf() # extract the acf
dim(Tz) # == c(nrow(Tz), ncol(Tz))
Tz # print method

# linear algebra methods
X <- matrix(rnorm(10), 5, 2)
Tz %*% X
t(X) %*% Tz
solve(Tz, X)
determinant(Tz) # log-determinant
```

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